Physics and Risk Management
What is Common?

Alain Bensoussan

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Probabilistic Theories replace Deterministic Theories

Probability theory was motivated by issues related to games of chance (Pascal, Fermat, Huygens, DeMoivre, Laplace)

Main Driving Force: The world is deterministic, but because of ignorance, probability theory is necessary.

*Note that the primary objective has been prediction and decision making not understanding physical laws.

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

From Newtonian Mechanics to Quantum Mechanics

- Newtonian Mechanics is completely deterministic, but the possibility of observational errors has been considered as complementary (Simpson, Bernoulli, Lagrange, Gauss, Legendre, Laplace)
- Scientific Change: Introducing probabilities into the science of mechanics itself
  - Statistical mechanics (Maxwell, Boltzmann, Gibbs)
  - Quantum Mechanics (Schrödinger, Heisenberg, Born, Bohr, Dirac, Von Neumann…)
- Source of randomness: Collision of particles (kinetic theory of gases, Brownian motion..)
- Heisenberg's uncertainty principle: Issue of measurement intrinsic dispersion of quantum mechanical particles

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Indeterminism Has Been Reluctantly Accepted

- The <u>Bohr-Einstein Debate:</u> **For Bohr, Classical Physics and Quantum theory are <u>irreconcilable</u>. The wave-particle duality prevents the description of physical objects in terms of classical concepts. The quantum of action attributes to any atomic process an individuality foreign to classical theories.**
- **An unambiguous definition of the state of a system is not possible because of interaction with measurement processes.**
- **Schrödinger has introduced the <u>wave function</u> Ψ associated to systems of particles, solutions of a wave equation (the Schrödinger equation), from which a lot of physical properties could be explained. The success of this model has been clear, in spite of the fact that the physical meaning of the wave function was not.**

The Bohr-Einstein Debate

- Bohr introduced a completely original point of view. Ψ has no physical meaning; $|\Psi|^2$ is the density of probability of the physical attributes of the particle (which is conceived in a classical sense), and we <u>cannot know more</u> than this probability density.
- The audacious aspect is that what we compute represents <u>our knowledge</u> and not the objective reality.
- This aspect is similar to the point of view of Risk Management.
- Einstein opposed strongly the Bohr-Born point of view: In a letter to Born (1920) Einstein writes:

  "That question of causality worries me a lot. Will the quantum absorption and emission of light ever be grasped in the sense of complete causality, or will there remain a statistical residue? I have to confess, that I lack the courage of a conviction. However I should be very, very loath to abandon <u>complete</u> causality…"

The Bohr-Einstein Debate

- Following an article of Bohr, Kramer's, Slater (1924) on a probabilistic approach to radiation, Einstein wrote to Born:

  "I cannot bear the thought that an electron exposed to a ray should by its <u>own free decision</u> choose the moment and the direction which it wants to jump away. If so, I'd rather be a cobbler or even an employee in a gambling house than a physicist."

- In another letter to Born, Einstein writes:

  "You believe in a God who plays dice, and I in complete law and order in a world which objectively exists."

- Einstein has contributed a lot to the development of statistical methods (Brownian motion). In fact, he was in favor of a frequency interpretation. So, $|\Psi|^2$ represented not the probability of a random particle, but the relative frequency of particles in certain conditions, in a set of particles. In this understanding Quantum Mechanics is just a branch of statistical mechanics.
- In fact, Einstein was looking for a more complete theory. There is famous paper (1935) EPR, Einstein-Podolsky-Rosen called "Can Quantum- Mechanical Description of Physical Reality be considered Complete?"
- He was groping for a radical departure from the conventional approach, analogous to the way the theory of general relativity supplanted Newton's theory of gravitation.

Risk Management

- In Risk Management, there are natural risks, but most risks are related to decisions of human beings.
- This is the source of indeterminism. The objective is to define relations calculating probabilities of relevant quantities. So the approach is close to that of Bohr-Born. The frequency interpretation is used for external random variables (surveys, public data…).
- For practical use of theories (in physics) and models (in risk management), namely for prediction, the approaches are very similar.

Models and Reality

- There is a pragmatist attitude which now prevails in Physics. It is symbolized by Gibbs in statistical mechanics, and by Bohr in Quantum Mechanics.
- Gibbs: "Statistical mechanics, like other physical theories, is an intellectual construction. Its value is in the ability to explain empirical laws."
- Bohr: "The entire formalism is to be considered as a tool for deriving predictions, of definite or statistical character, as regards information obtainable under experimental conditions described in classical terms."

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Models and Reality

- "There is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature."
- This pragmatist attitude is prevalent in Risk Management. The word model instead of theory emphasizes this point.
- <u>Consequence:</u> in R.M. but also in Physics, simplicity may be preferable to unnecessary details, which may obscure the understanding.

  Importance of the central limit theorem (in R.M.)

  Importance of the concept of equilibrium (in statistical Mechanics)

Hidden Variables

This concept has been introduced in Quantum Mechanics by physicists, who rejected the probabilistic interpretation. They tried to demonstrate that the theory in spite of its spectacular success is only a provisional approximation to a deeper scientific truth.

It was not promoted by Einstein, who wanted “a supremely interesting theory, by which I hope to overcome the present mystic of probability and the abandonment of the concept of reality in physics.”

Hidden Variables

So, the Hidden Variables approach is more a refinement, than a revolutionary theory. David Bohm characterized them as "a further set of variables, describing the state of new kinds of entities existing in a deeper sub quantum mechanical level and obeying qualitatively new types of individual laws".

Though "hidden," at present, these variables may "be revealed in detail when

Hidden Markov Models

- **In stochastic control, this term is identical to P<u>artially Observed Stochastic Dynamical Systems</u>. The observation process does not see the state of the system directly. It is hidden in the observations. The whole issue is to <u>estimate</u> the state from the observations.**
- **If the hidden variable does not depend on time, like the initial state, then a <u>learning</u> procedure may lead to a complete identification.**
- **This approach is not used in physics, probably because first it is a conceptual issue, and second because of a difference of scale, the observation process does not carry information on the hidden variables.**

The Role of Information

- In R.M. information plays an essential role. It is used to reduce uncertainties and to mitigate risks. Statistical inference and Bayesian Techniques are the driving forces.
- Information is often a matter of cost. So, a natural problem is to optimize the level of information which is needed, by a cost-benefit analysis.
- In statistical mechanics, a somewhat opposite situation takes place in the context of systems of particles, which reach an equilibrium. Equilibrium can occur when there exist a large number of degrees of freedom.
- The Maximum Entropy Principle (MER) is a justification of equilibrium. It corresponds to maximum uncertainty. In Bayesian techniques, MER appears in the choice of prior probabilities.

Networks and Queuing Systems

- **Queuing systems constitute a commonly used model in Operations Management and Supply Chain issues.**
- **Equilibrium is generally considered. It is a concept different from that of statistical mechanics, since the number of degrees of freedom is smaller. However, important laws have been derived. They relate expected values of variables and can be compared to the macroscopic laws of classical physics.**

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Objective and Subjective Probabilities

- <u>Objective Probabilities</u>: Under this point of view, probabilities are external, real, fixed. They are theoretical entities, comparable to mass or force.
- Objective probabilities are <u>observable</u>. One possibility to attain probabilities empirically is to perform time averages and use Ergodic Theory. In economic systems, the "frequentist" approach to probability is the most popular.
- Thanks to the central limit theorem, statistical inference has become a very powerful methodology

Subjective Probabilities

- **Under this point of view, it is not necessary to give an objective interpretation to probabilities. Probabilities represent <u>degrees of belief,</u> formulated because of ignorance with respect to the exact data. The system itself can be deterministic. The value of subjective probabilities lies in the quality of prediction.**

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Objective and Subjective Probabilities in Risk Management

- The concept of subjective probabilities is widely accepted in Risk Management. Expert opinions play an essential role in the modeling. One uses the terminology: variability and uncertainty.
- Variability: Relates to randomness which cannot be reduced by acquiring information; It is intrinsic.
- Uncertainty: relates to randomness which can be reduced by acquiring information. It may concern parameters of "structural" distributions (objective in Physics). Updating techniques are prevalent (Bayesian approach).

Quantum Mechanics and Markov Processes

- An interpretation of quantum mechanics as a classical theory of stochastic processes is possible. This is due to the similarity of Schrödinger equation and equations of the theory of diffusion processes.
- There is a connection between Feynmann integrals, Markov processes and Schrödinger equation.
- This analogy is mathematical. There are no empirical facts corroborating the existence of microphysical particles performing some kind of Brownian motion.
- On the other hand these Markov processes are very similar to those introduced in the models of stochastic control.

The Value Function

- The value function is the key entity of interest in Risk Management problems. It is solution of a non linear diffusion equation, called Bellman equation, or Hamilton Jacobi Bellman Equation.
- This plays the role of Schrödinger equation of Quantum Mechanics and is more directly related to the Markov process interpretation of the Schrödinger equation.
- Optimal stopping time problems are solution of Variational Inequalities. These V.I. are identical to those encountered in modeling displacement in elasto plastic structures.

Path Integrals in Physics and Finance

- Path integrals were developed by Feynman as a technique used in his Nobel-prize winning work related to relativistic quantum mechanics. Further work by Kac and others, along with many applications to physics followed.
- Any diffusion equation has a path integral solution. Brownian motion has been introduced by Bachelier in 1900 (earlier then the work of Einstein) to represent the uncertainty resulting from many individual effects. In modern finance, standard pricing models for interest rates, stock prices, exchange rates… involve diffusion equations. The path integral approach has proved useful as a realistic tool in finance.

ICDRiA INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Risk Management: Difficulties of Implementation

- **Risks in real life are considered reluctantly. The belief in determinism which has prevailed in physics is replaced by a refusal to look at risks in a scientific manner. Either one demands no risk, which is impossible or too costly, or one ignores risks which is very dangerous.**
- **It is very difficult to move from numbers to probabilities. First, the probabilities are not known and second the interdependencies are hard to take into account. Scientific Risk Management has been extremely powerful in finance, providing ingenious tools to mitigate risks. The challenge is to extend this approach to project risk management.**
- **Also there remain risks which cannot be quantified. There are new risks for which there is no past experience, and we may have a feeling of protection which is not justified.**

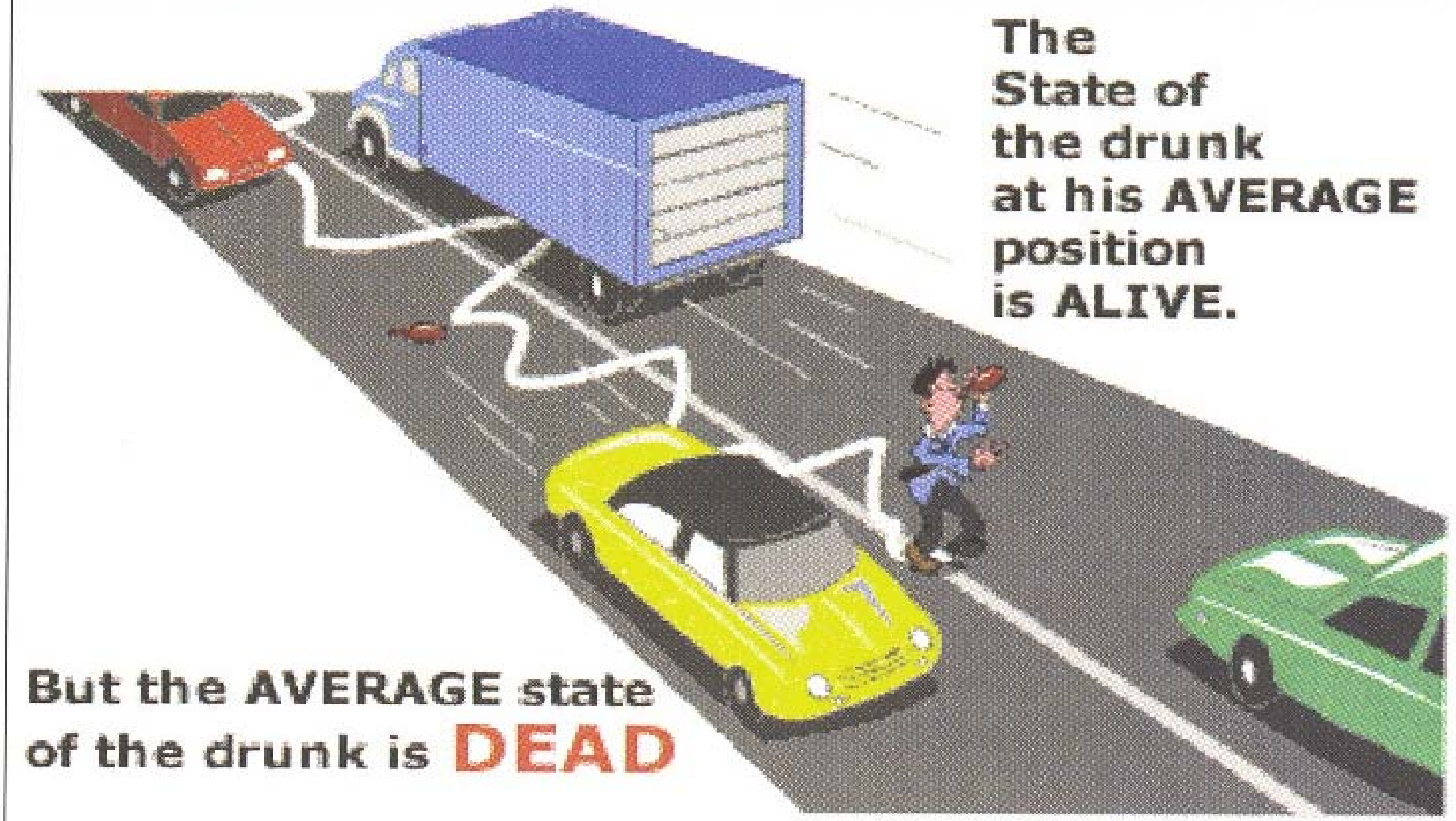

Figure 1: *A drunk staggering down the middle of a busy highway illustrates a sobering example of the flaw of averages [9].*

ICDRiA
INTERNATIONAL CENTER FOR DECISION AND RISK ANALYSIS

Philosophical, Religious, Political Aspects

- **Probabilistic and risk consideration raise a lot of unsolved questions:**

- What is the objective reality?

For Einstein (Physics and Reality 1936), although real existence of objects is declared by humans, we are capable to access it.

"The eternal mystery of the world is its comprehensibility."

- The subjective point of view:

We have only access to some knowledge and never to the complete reality. The knowledge changes with information. So, physics is not human-independent. Is the physicist investigating nature or his own investigations?

- Is reality merely a relation between substances, or do substances have properties independently of their relations to other substances?

- Is quantum mechanical indeterminism harmonizing with the belief in God?

- Can Risk Management be based on purely rational grounds?

Concluding Remarks

- It is interesting to notice that the extremely high level debates among scientific giants were occurring at very dark political times, in Germany, in the Soviet Union (the theory of Bohr has been considered as irreconcilable with dialectical materialism), but also in the U.S. (Bohm has been suspended from his position under the McCarthy times and left to Brazil).
- Today a pragmatist vision prevails in science, with a shift towards ethical debates.
- This pragmatist approach does not reflect necessarily a general tendency of mankind's orientation.